Exam 2   **Set Theory**   **22–12–2014**

1. Translate and prove:

    (a) There is a real number x such that $x = x^2$.

    (b) Not all integers x satisfy $x + 1 > 2x$.

2. Use contradiction to prove that $\sqrt[3]{2}$ is irrational.

3. Use induction to prove the following formula for all $n \in \mathbb{N}$.

$$1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \cdots + n \cdot n! = (n+1)! - 1$$

4. Let $R = \left\{ (a, b) \in \mathbb{N} \times \mathbb{N} \mid a + b \text{ is odd} \right\}$. True or false?

    (a) Is R reflexive?

    (b) Is R symmetric?

    (c) Is R anti-symmetric?

    (d) Is R transitive?

5. Let $R = \left\{ (x, y) \in \mathbb{N} \times \mathbb{N} \mid x - y = 7k \text{ for some } k \in \mathbb{Z} \right\}$.

    (a) Prove that R is an equivalence relation.

    (b) What are the elements in the equivalence class [20]?

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