Philadelphia University

Department of Basic Sciences

Exam 2 **Set Theory** **30–04–2014**

1. Use contradiction to prove that $\sqrt[5]{2^2}$ is an irrational number.
2. Translate and prove:
   (a) there is a natural number x such that $x^2 = 5x - 6$.
   (b) not all rational numbers x satisfy $x^2 \geq x$.
3. Use induction to prove that $5^{2n} - 4^n$ is a multiple of 3 for all $n \in \mathbb{N}$.
4. Let $R = \{(a, b) \in \mathbb{N} \times \mathbb{N} \mid a + b > 2\}$. True or false?
   (a) Is R reflexive?
   (b) Is R symmetric?
   (c) Is R anti-symmetric?
   (d) Is R transitive?
5. Let $R = \{(a, b) \in \mathbb{N} \times \mathbb{N} \mid a \bmod b = 0\}$. Prove that R is a partial order relation.

–Amin Witno