Final Exam   **Set Theory**   **17–01–2013**

Part 1. Short Answer, 2 points each.

1. The proposition $p \to (q \to r)$ is equivalent to: (choose one)
   (a) $q \to (p \to r)$ (b) $r \to (q \to p)$ (c) $(p \to q) \to r$ (d) $(q \to r) \to p$

2. If $A \subseteq B$ then $A \oplus B =$ (choose one)
   (a) $A \cup B$ (b) $A \cap B$ (c) $A - B$ (d) $B - A$

3. Let $S = \{x \in \mathbb{Q} \mid 2x \in \mathbb{Z}\}$ and $T = \{x \in \mathbb{R} \mid x^2 \leq 4\}$. Find the elements in $S \cap T$.

4. Let $A = \{1, 2, 3, 4\}$. Give one example of a relation R on A that is symmetric and transitive, but not anti-symmetric.

5. Let $A = \{x \in \mathbb{N} \mid x < 20\}$. The set $R = \{(a, b) \in A \times A \mid a \bmod 3 = b \bmod 3\}$ is an equivalence relation on A. Find the elements of the equivalence class $[5]$.

6. True or false: The relation $R = \{(a, b) \in \mathbb{N} \times \mathbb{N} \mid a \leq b\}$ is a total order on $\mathbb{N}$.

7. Let $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = x^2 + 1$. Find $f^{-1}([0, 10])$.

8. Which function $f : \mathbb{Z} \to \mathbb{Z}$ is a bijection? (choose one)
   (a) $f(x) = 2n - 1$ (b) $f(n) = 2n$ (c) $f(n) = 2n + 1$ (d) $f(n) = n + 1$

9. True or false: If $|A| = \aleph_0$ and $|B| = \aleph_0$, then $|A \cup B| = \aleph_0$.

10. Which set is countable? (choose one)
    (a) $\mathbb{Q}$ (b) $\mathbb{R}$ (c) $P(\mathbb{N})$ (d) $P(\mathbb{R})$

Part 2. Complete Solution, 5 points each.

1. Let $x \in \mathbb{R}$. Prove that if $x^2 - x + 1$ is irrational, then x is irrational.

2. Use induction to prove that $3^n < n!$ for all integer $n \geq 7$.

3. $A = \{1, 2, 3\}$. Let R be the relation on $P(A)$ given by $R = \{(X, Y) \mid X \subseteq Y\}$. Prove that R is a partial order relation.

4. Let S be a finite set. Prove that $|\mathbb{N} \cup S| = \aleph_0$.

–Amin Witno