**Set Theory [Exam 1] 2–4–2006**

Each problem is worth 5 points.

1. (a) Draw the truth table for $(\neg p \to q) \leftrightarrow (\neg q \to p)$.

   (b) Write the negation of $p :$ "for all real numbers $x$, $x^2 > 2x$".

   (c) Let $P(x,y) : x^2 > y^3$. Find the values of $\exists y \forall x P(x,y)$ and $\forall y \exists x P(x,y)$.

2. (a) Prove that there is an integer $n$ such that $n \bmod 3 = 2$ and $n \bmod 4 = 3$.

   (b) Prove that there is a unique natural number $n$ such that $n^2 = n$.

3. Prove that $n^2 - 2n + 5$ is even if and only if $n$ is odd.

4. Prove by contradiction that $\sqrt[3]{2}$ is irrational.

–Amin Witno–