

All answers must be in reduced fractions or decimal with 4 significant digits.

1. (6pt) A coin is tossed 8 times.  
Let  $A = \{\# \text{ Heads} = 5\}$  and  $B = \{\text{the first is Tails}\}$ .
  - (a) Compute  $P(A)$
  - (b) Compute  $P(B)$
  - (c) Compute  $P(A \cap B)$
  - (d) Compute  $P(A | B)$
  - (e) Compute  $P(A \cup B)$
  - (f) Prove  $A$  and  $B$  are dependent or independent.
2. (2pt) An airline flight has a probability 23% of getting delayed when it is raining, and 11% when not raining. The probability that tomorrow will rain is 88%. Find the probability that this flight will be delayed tomorrow.
3. (2pt) Assume that 80% of the Samsung mobile phones in the country come from China, 8% from Malaysia, and 12% from India. From China, an average 2.2% of the phones are defective, from Malaysia 2.9%, and from India 4.7%. Given that a phone is found defective, what is the probability it came from India?
4. (2pt) Given the probability density function  $f(x)$ . Compute  $P(X > \frac{3}{4})$

$$f(x) = \begin{cases} 5x^4 & \text{for } 0 < x < 1 \\ 0 & \text{else} \end{cases}$$

5. (1pt) Given the discrete distribution function  $F(x)$ . Compute  $P(X < 3)$

$$F(x) = \begin{cases} 0 & \text{for } x < 1 \\ \frac{1}{4} & \text{for } 1 \leq x < 3 \\ \frac{3}{5} & \text{for } 3 \leq x < 5 \\ 1 & \text{for } x \geq 5 \end{cases}$$

6. (2pt) Given the probability density function  $f(x)$ . Find  $F(x)$ .

$$f(x) = \begin{cases} \frac{1}{4\sqrt{x}} & \text{for } 1 < x < 9 \\ 0 & \text{else} \end{cases}$$

7. (2pt) Given the distribution function  $F(x)$ . Compute  $P(4.5 \leq X \leq 6)$

$$F(x) = \begin{cases} 1 - \frac{9}{x^2} & \text{for } x \geq 3 \\ 0 & \text{else} \end{cases}$$

8. (2pt) Given the joint probability distribution  $f(x, y) = k \binom{6}{x+y}$  where  $x \in \{3, 4\}$  and  $y \in \{1, 2\}$ . Find  $k$

9. (3pt) Given the joint probability density function  $f(x, y)$ . Find  $k$ .

$$f(x, y) = \begin{cases} kxy & \text{for } x, y > 0; x + y < 1 \\ 0 & \text{otherwise} \end{cases}$$

10. (3pt) Given the joint probability density function  $f(x, y)$ . Find  $F(x, y)$

$$f(x, y) = \begin{cases} x + y & \text{for } 0 < x, y < 1 \\ 0 & \text{otherwise} \end{cases}$$

11. (3pt) Given the joint probability density  $f(x, y)$ . Compute  $P(X < 1; Y \leq \frac{1}{2})$

$$f(x, y) = \begin{cases} 2 & \text{for } x, y > 0; x + y < 1 \\ 0 & \text{otherwise} \end{cases}$$

12. (3pt) Given the joint distribution function  $F(x, y)$ . Compute  $P(X + Y < 1)$

$$F(x, y) = \begin{cases} 1 - e^{-x} - e^{-y} + e^{-x-y} & \text{for } x, y > 0 \\ 0 & \text{otherwise} \end{cases}$$