Final Exam **Probability Theory** **13/02/2023**

Each problem is worth 4 points.

1. Two dice are rolled. Let $A = \{\text{the sum is} > 6\}$. Let $B = \{\text{both are odd}\}$. Compute $P(A \cup B)$.

2. Given the distribution function $F(x)$, find $P(6 < X < 9)$.

$$F(x) = \begin{cases} 1 - \frac{9}{x^2} & \text{for } x \geq 3 \\ 0 & \text{for } x < 3 \end{cases}$$

3. Given the joint distribution function $F(x, y)$, find $P(1 < X < 2;\ Y \leq 2)$.

$$F(x,y) = \begin{cases} 1 - e^{-x} - e^{-y} + e^{-x-y} & \text{for } x, y > 0 \\ 0 & \text{otherwise} \end{cases}$$

4. Given the joint probability density function $f(x, y)$. Compute the conditional density of Y given $(X = \frac{1}{2})$.

$$f(x,y) = \begin{cases} \frac{1}{5}(2x + y) & \text{for } 0 < x < 2;\ 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

5. Given the joint probability density function $f(x, y)$. Find $P(X, Y < \frac{3}{2})$.

$$f(x,y) = \begin{cases} \frac{4}{3}xy & \text{for } 0 < x < 1;\ 1 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

6. Given the joint probability density function $f(x, y)$.

$$f(x,y) = \begin{cases} 4xy & \text{for } x, y > 0;\ x + y < 1 \\ 0 & \text{otherwise} \end{cases}$$

   Write the double integral for $P(X > Y)$. (ONLY the integral, do not compute).

7. Compute the covariance σ_{XY} given the joint probability density function

$$f(x,y) = \begin{cases} x + y & \text{for } 0 < x, y < 1 \\ 0 & \text{otherwise} \end{cases}$$

8. Given the discrete uniform distribution $f(x) = \frac{1}{4}$ with domain $x \in \{-1, 0, 1, 2\}$. Compute μ and σ^2

9. Given that $\sigma_X^2 = 3$, $\sigma_Y^2 = 4$, $\sigma_Z^2 = 5$ and $\sigma_{XY} = 3$, $\sigma_{XZ} = -2$, $\sigma_{YZ} = 1$. Let $W = X + 2Y - 3Z$. Compute the variance σ_W^2.

10. Given the Pareto distribution $f(x) = \frac{2}{x^3}$ with domain $x \in (1, \infty)$. Compute μ and σ^2.

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