# Philadelphia University <br> Department of Basic Sciences 

Exam 2
Probability Theory
30-12-2019

1. (2pt) Find $k$ for the given joint probability density function

$$
f(x, y)=\left\{\begin{array}{cl}
k(x+y) & \text { for } 0<x<1 ; 0<y<2 \\
0 & \text { otherwise }
\end{array}\right.
$$

2. (3pt) Compute $E(X)$ and $E(4 X+3)$ for the given probability distribution

| $x$ | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{2}$ | $\frac{1}{4}$ |

3. (3pt) Compute $\mu$ and $\sigma^{2}$ for the given probability density function

$$
f(x)= \begin{cases}\frac{x}{2} & \text { for } 0<x<2 \\ 0 & \text { otherwise }\end{cases}
$$

4. (6pt) Given the joint probability density function

$$
f(x, y)=\left\{\begin{array}{cl}
60 x^{2} y & \text { for } 0<x, y<1 ; x+y<1 \\
0 & \text { otherwise }
\end{array}\right.
$$

(a) Compute the marginal density of $X$
(b) Prove $X$ and $Y$ independent or dependent
(c) Compute the conditional density of $Y$ given $X=\frac{1}{2}$
5. (6pt) Given the joint probability density function

$$
f(x, y)=\left\{\begin{array}{cl}
2 x(x-y) & \text { for } 0<x<1 ;-x<y<x \\
0 & \text { otherwise }
\end{array}\right.
$$

(a) Compute $P\left(X \geq \frac{1}{2} ; Y<1\right)$
(b) Compute $P\left(X<1 ; Y<\frac{1}{2}\right)$

