PHILADELPHIA UNIVERSITY

DEPARTMENT OF BASIC SCIENCES

Exam 1   **Probability Theory**   **27–11–2019**

1. (*) In a supermarket there are 89 apples, and 11 of them are bad. If we randomly pick 2 apples, compute the probability that both are bad.

2. (*) A coin is tossed 7 times. Compute the probability of exactly 2 heads, given that the first outcome is tail.

3. (**) Two dice are rolled. Compute the probability either the two numbers are equal, or the sum is 8.

4. (**) A coin is tossed 5 times. Let A be the event that we get at least 4 heads. Let B be the event that the first is tail. Prove A and B are independent or dependent.

5. (***) My probability to pass this exam is 77% if I study, or 48% if I do not study. The probability I have time to study is 55%. Compute the probability I pass this exam.

6. (***) Philadelphia University has 69% of their buses from their own, 11% rented from Company A, and 20% rented from Company B. From their own buses, 7% have broken conditioning. From Company A, 47% have broken conditioning, and from Company B, 21% has broken conditioning. Given that a bus has broken conditioning, what is the probability it comes from Company A?

7. (*) Given the probability distribution $f(x) = k\binom{3}{x}$, where $x \in \{0, 1, 2\}$, find the value of k.

8. (**) Given the probability density function (p.d.f.) $f(x)$, find the value of c.
$$f(x) = \begin{cases} cx^3 & \text{for } x \in (0,2) \\ 0 & \text{for } x \notin (0,2) \end{cases}$$

9. (*) Given the distribution function $F(x)$, compute $P(X = 1)$.
$$F(x) = \begin{cases} 0 & \text{for } x < -1 \\ \frac{1}{4} & \text{for } -1 \leq x < 1 \\ \frac{2}{3} & \text{for } 1 \leq x < 3 \\ 1 & \text{for } x \geq 3 \end{cases}$$

10. (*) Given the probability density function (p.d.f) $f(x)$, compute $P(1 \leq X \leq 4)$.
$$f(x) = \begin{cases} \frac{1}{6\sqrt{x}} & \text{for } x \in (0,9) \\ 0 & \text{for } x \notin (0,9) \end{cases}$$

11. (**) Find the distribution function $F(x)$ for Problem (10).

12. (*) Given the distribution function $F(x)$, compute $P(X \leq 2)$.
$$F(x) = \begin{cases} 1 - (1+x)e^{-x} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$$

<div align="right">–Amin Witno</div>