# Philadelphia University
## Department of Basic Sciences

**Exam 2** **Problem Solving** **29–12–2014**

1. (a) Count how many integer solutions with $x_i \geq 2$ in the following equation.

$$x_1 + x_2 + x_3 + x_4 + x_5 = 50$$

   (b) Evaluate the sum $S$.

$$S = \binom{50}{46} + 3\binom{50}{47} + 3\binom{50}{48} + \binom{50}{49}$$

2. Find the formula for $S_n$ given that $S_0 = 2$ and $S_1 = 1$, where for $n \geq 2$,

$$S_n = 3S_{n-1} + 4S_{n-2}$$

3. Find the formula for the sum

$$\sum_{k=0}^{n} \binom{n}{k} k^2$$

by differentiating the binomial expansion for $(1+x)^n$.

4. Write the identity for the following pattern and prove it.

$$\begin{aligned} 1\cdot 1 &= 1 \\ 1\cdot 1 + 1\cdot 2 &= 3 \\ 1\cdot 1 + 2\cdot 2 + 1\cdot 4 &= 9 \\ 1\cdot 1 + 3\cdot 2 + 3\cdot 4 + 1\cdot 8 &= 27 \\ 1\cdot 1 + 4\cdot 2 + 6\cdot 4 + 4\cdot 8 + 1\cdot 16 &= 81 \\ \cdots &= \cdots \end{aligned}$$

5. Find the numbers $a$ and $b$ in the following formula, without proof.

   (a)

$$\binom{n}{1}^2 - \binom{n}{1} - \binom{n}{2} = \binom{a}{b}$$

   (b)

$$\sum_{m=6}^{n} \binom{m}{6} = \binom{a}{b}$$

–Amin Witno