Philadelphia University
Department of Basic Sciences

Final Exam **Numerical Analysis** **22–06–2022**

1. (6 points) Given $f(x) = x^4 - 2x^3 + 2x + 1$ with $p_0 = -1$, use Horner method to find p_2 as a rational number.

2. (6 points) Let $f(x) = \sin(\ln x)$. (a) Approximate $f(2.2)$ using $P(2.2)$, where $P(x)$ is the Lagrange polynomial degree 2 with $x_0 = 2$, $x_1 = 2.4$, and $x_2 = 2.6$.
   (b) Find the actual error.
   Lagrange polynomial: $P(x) = \sum f(x_k)L_k(x)$, where $L_k(x) = \prod_{i \neq k} \frac{(x-x_i)}{(x_k-x_i)}$

3. (6 points) Use Simpson's rule for $\int_0^{0.1} \sqrt{1+x}\, dx$, and find the error bound.
   Simpson's rule: $\int_{x_0}^{x_2} f(x)dx = \frac{h}{3}[f(x_0) + 4f(x_1) + f(x_2)] - \frac{h^5}{90} f^{(4)}(t)$

4. (6 points) Approximate $\int_0^2 x^2 e^{-x^2} dx$ with $h = 0.5$ using (a) Composite Trapezoidal rule (b) Composite Simpson's rule.

5. (8 points) (a) Prove that $P_3(x) = x^3 - \frac{3}{5}x$ is Legendre polynomial degree 3.
   (b) Approximate $\int_1^{1.6} \frac{2x}{x^2-4} dx$ using Gaussian Quadrature with $n = 3$.
   (c) Find the actual error.
   Gaussian quadrature: $\int_{-1}^{1} f(x)dx \approx \frac{1}{9}[5f(-\sqrt{0.6}) + 8f(0) + 5f(\sqrt{0.6})]$, where $\int_a^b f(x)dx = \int_{-1}^{1} f\left(\frac{(b-a)t+(b+a)}{2}\right)\frac{(b-a)}{2}dt$

6. (8 points) (a) Use Euler method for the initial value problem $y' = 1 + (t-y)^2$ on the interval $3 \leq t \leq 4$ with $y(3) = 2.5$ and $h = 0.5$
   (b) Show that $y(t) = t + \frac{1}{1-t}$ is exact solution.
   (c) Find the actual error for $y(4)$.

–Amin Witno