Philadelphia University

Department of Basic Sciences

Final Exam **Numerical Analysis** **21–01–2013**

Write complete solution for each problem.

1. Consider the function $f(x) = \sin(\ln x)$.
   (a) Construct Lagrange interpolation polynomial of degree two for $f(x)$ using $x_0 = 2$, $x_1 = 2.4$, and $x_2 = 2.6$.
   (b) Use the result in (a) to approximate $f(2.2)$.
   (c) Compute the actual error for the result in (b).

2. Approximate the definite integral
$$\int_0^{0.1} \sqrt{1+x}\, dx$$
using the following methods, and find the error bound for each result.
   (a) Trapezoidal rule
   (b) Simpson's rule
   (c) Simpson's Three-Eighths rule

3. Consider the differential equation $y' = 1 + y/t + (y/t)^2$ with initial value $y(1) = 0$.
   (a) Approximate $y(1.4)$ using Euler's method with $h = 0.2$.
   (b) Compute the actual error, given the exact solution $y(t) = t\tan(\ln t)$.

4. Use Taylor's method of order two to approximate the solution to the differential equation $y' = \sin t + e^{-t}$ with initial value $y(0) = 0$, in the interval $0 \le t \le 1$ using $h = 0.5$.

5. Consider the differential equation $y' = y - t^2 + 1$ with the given initial value $y(0.1) = 0.657414$. Use the Runge-Kutta method of order four to approximate $y(0.2)$ using $h = 0.1$.

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