Philadelphia University
Department of Basic Sciences

Final Exam **Computational Number Theory** **07–02–2008**

1. Let $n = 10t + u$. Then $19 \mid n$ if and only if $19 \mid (t + 2u)$. Illustrate this divisibility test with $n = 20080131$. What is your conclusion?
2. Express the fraction $\frac{250}{472}$ using a finite continued fraction.
3. We are applying the Quadratic Sieve method with $n = 897$.

	30^2	43^2	60^2	90^2	109^2
2					
3					
5					
7					
11					

   Complete the table and finish the algorithm.

4. Illustrate Miller-Rabin test with $n = 1201$ and $a = 3$. What is your conclusion?
5. Apply Euler test for $n = 529$ with $a = 2$. What is your conclusion?
6. (a) What is a Carmichael number? Show why the number 2465 is Carmichael.
   (b) What is a perfect number? Show why the number 496 is perfect.
   (c) What is a triangular number? Show why the number 56616 is triangular.
7. Suppose that n is a Fermat pseudoprime base 2.
   (a) Prove that $2^n - 1$ is composite.
   (b) Prove that $2^n - 1$ is a Fermat pseudoprime base 2.
8. Let F_n denote a Fermat number. Prove the following statements.
   (a) If $a^{\frac{F_n - 1}{2}} \equiv -1 \pmod{F_n}$ then F_n is prime.
   (b) The converse is true if $x^2 \equiv a \pmod{F_n}$ has no solution.
   (c) The converse is false if $x^2 \equiv a \pmod{F_n}$ has a solution.

–Amin Witno