Final Exam **Number Theory** **19/06/2023**

1. Each question is worth 2 points.
   (a) Find $|g^{84}|_{19}$ given that g is primitive root mod 19.
   (b) Find all the primitive roots mod 9.
   (c) Count how many primitive roots mod 353 (prime).
   (d) Find all the NR mod 11.
   (e) Determine -8 is QR or NR mod 37.
2. (4 points) Use Fermat's little theorem to prove that $589 \mid n^{91} - n$ for all $n \in \mathbb{Z}$. (Note $589 = 19 \times 31$)
3. (4 points) Solve the root mod congruence $x^{13} \equiv 2 \pmod{23}$.
4. (4 points) Compute $4^{2691} \,\%\, 35$ using Euler's theorem.
5. (a) (2 points) Evaluate $|7|_{22}$ by completing the following table.

k	1	2	3	4	5	6	7	8	9	10
$7^k \,\%\, 22$	7									

   (b) (4 points) Use the table to solve the discrete log problem $19^x \equiv 21 \pmod{22}$.
6. (4 points) Determine solutions exist or not exist using the Legendre symbol.

$$2x^2 + 9x + 21 \equiv 0 \pmod{257}$$

7. (4 points) Find the four solution classes of $x^2 \equiv 67 \pmod{187}$. (Note $187 = 11 \times 17$)
8. (4 points) Prove that $\left(\frac{-2}{p}\right) = +1$ if and only if the prime $p \equiv 1$ or $3 \pmod 8$.