Midterm Exam **Number Theory** **10/05/2023**

1. (3 points) Prove Euclid's Lemma: If $x \mid yz$ and $\gcd(x, y) = 1$, then $x \mid z$
2. (3 points) Find all the integer solutions for $153x - 39y = 15$
3. (2 points) Let p be a prime. Prove that if $p \mid m^3$, then $p^3 \mid m^3$
4. (2 points) Count how many divisors of the number 1728
5. (3 points) Use Fermat factorization algorithm to factor $n = 10873$
6. (2 points) Compute $7^{-1} \ \% \ 25$
7. (4 points) Solve the system of linear congruences $\begin{cases} x \equiv 15 \pmod{25} \\ x \equiv 5 \pmod{9} \end{cases}$
8. (3 points) Prove the theorem: If $a \equiv b \pmod{n}$, then $n \mid a - b$
9. (4 points) Compute $43! \ \% \ 47$ using Wilson's theorem.
10. (4 points) Use SSA to compute $7^{98} \ \% \ 11$
11. (Bonus 2 points) Prove that if $a \equiv b \pmod{n}$, then $\gcd(a, n) = \gcd(b, n)$