Final Exam **Number Theory** **05/02/2023**

1. (4 points) Compute 9^{56} % 11 using SSA.
2. (4 points) Compute 9^{3723} % 35 using Euler's theorem.
3. (2 points) Evaluate $\phi(5616)$.
4. (4 points) Solve the root mod congruence $x^{13} \equiv 2 \pmod{23}$.
5. (2 points) Find $|3^{84}|_{19}$ given that 3 is primitive root mod 19.
6. (2 points) Find all the primitive roots mod 22.
7. (4 points) Solve the discrete log problem $19^x \equiv 21 \pmod{22}$.
   You may use your table from Question (6).
8. (2 points) Count how many primitive roots exist mod 541 (prime).
9. (4 points) Find the 4 solution classes of $x^2 \equiv 37 \pmod{77}$.
10. (2 points) Prove that if a prime $p \equiv 3 \pmod 4$, then -1 is NR mod p.
11. (2 points) Determine 32 is QR or NR mod 101.
12. (4 points) Evaluate the Legendre symbol $\left(\frac{101}{313}\right)$.
13. (4 points) Prove the theorem: Let $S = \{g, g^2, \ldots, g^{\phi(n)}\}$. Prove that S is RRS mod n if and only if g is primitive root mod n.
14. (Bonus 3 points) Prove that if g is a primitive root mod a prime $p > 2$, then $\left(\dfrac{g}{p}\right) = -1$.

–Amin Witno

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