# Philadelphia University Department of Basic Sciences 

Final Exam

## Number Theory

26-05-2019

1. $\left(7\right.$ points) Find all the solutions of $x^{13} \equiv 2(\bmod 23)$.
2. ( 7 points) Find all the solutions of $5^{x} \equiv 3(\bmod 11)$.
3. ( 8 points) Find all the solutions of $x^{2} \equiv 60(\bmod 77)$.
4. (8 points) Evaluate the Legendre symbol $\left(\frac{-66}{191}\right)$.
5. (10 points) Solve 2 problems from the following 3:
(a) Prove that if $k$ is even, then $\phi(2 k)=2 \phi(k)$.
(b) Prove that if $a^{8} \equiv-1(\bmod 17)$, then $a$ is primitive root $\bmod 17$.
(c) Prove that if $p \% 8=3$, then $\left(\frac{-2}{p}\right)=+1$
