## PHILADELPHIA UNIVERSITY DEPARTMENT OF BASIC SCIENCES

Final Exam

## Number Theory

26 - 05 - 2019

- 1. (7 points) Find all the solutions of  $x^{13} \equiv 2 \pmod{23}$ .
- 2. (7 points) Find all the solutions of  $5^x \equiv 3 \pmod{11}$ .
- 3. (8 points) Find all the solutions of  $x^2 \equiv 60 \pmod{77}$ .
- 4. (8 points) Evaluate the Legendre symbol  $\left(\frac{-66}{191}\right)$ .
- 5. (10 points) Solve 2 problems from the following 3:
  - (a) Prove that if k is even, then  $\phi(2k) = 2\phi(k)$ .
  - (b) Prove that if  $a^8 \equiv -1 \pmod{17}$ , then *a* is primitive root mod 17.

(c) Prove that if p % 8 = 3, then  $\left(\frac{-2}{p}\right) = +1$ 

-Amin Witno