

PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

Exam 2

Number Theory

08-05-2019

Choose 5 problems out of 6. No bonus.

1. This problem has 4 parts:
 - (a) Evaluate $|8|_{13}$.
 - (b) If g is a primitive root mod 25, evaluate $|g^{15}|_{25}$.
 - (c) Count how many primitive roots exist mod 31.
 - (d) Find all the primitive roots mod 19 using the table provided below.

k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$2^k \% 19$	2	4	8	16	13	7	14	9	18	17	15	11	3	6	12	5	10	1

2. Evaluate $11^{70} \% 100$ using successive squaring algorithm.
3. Find all the integer solutions of $x^{13} \equiv 9 \pmod{32}$.
4. This problem has 2 parts:
 - (a) Prove that 3 is a primitive root mod 14.
 - (b) Use (a) to find all the integer solutions of $9^x \equiv 11 \pmod{14}$.
5. Prove that $m^{61} \equiv m \pmod{155}$ for all integers m . (Use the Chinese remainder theorem and Fermat's little theorem.)
6. Prove that $\phi(a^2) = a\phi(a)$ for all positive integers a .

-Amin Witno