# Philadelphia University

## Department of Basic Sciences

**Exam 1** **Number Theory** **03–04–2019**

Choose 5 problems out of 6 and write complete solutions. No bonus.

1. Find the general solution of the linear equation $406x + 350y = 42$.
2. This problem has 2 parts.
   (a) Determine prime or composite using Trial Division, for $n = 559$. If composite, write the factorization.
   (b) Apply Fermat Factorization Method for $n = 5917$.
3. Solve the system of two linear congruences: $\begin{cases} x \equiv 19 \pmod{25} \\ x \equiv 23 \pmod{32} \end{cases}$
4. Prove that $5 \mid x^5 - x$ for all $x \in \mathbb{Z}$.
5. Prove the theorem: Let $\gcd(b, k) = 1$. If $b \mid f$ and $k \mid f$, then $bk \mid f$.
6. This problem has 2 parts.
   (a) What is our definition of $a \equiv b \pmod{n}$?
   (b) Prove the theorem: $a \equiv b \pmod{n}$ if and only if $n \mid a - b$.

–Amin Witno