Final Exam **Number Theory** **23–01–2019**

1. Solve the linear equation $48x - 28y = 32$.

2. Solve the congruence $x^5 \equiv 3 \pmod{38}$.

3. Solve the congruence $11^x \equiv -4 \pmod{25}$ using the given table of $2^k \,\%\, 25$.

k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2^k	2	4	8	16	7	14	3	6	12	24	23	21	17	9	18	11	22	19	13	1

4. Solve the congruence $x^2 \equiv 37 \pmod{77}$.

5. Evaluate the Legendre symbol $\left(\frac{198}{283}\right)$.

6. Prove the theorem: If $a^m \,\%\, n = 1$, then $|a|_n \mid m$.

7. Let $p > 2$ be a prime number. Prove that if m is a primitive root mod p, then m is a quadratic non-residue (NR) mod p.

8. Prove that if a prime $p \,\%\, 8 = 3$, then $\left(\frac{-2}{p}\right) = +1$.

–Amin Witno