Final Exam **Number Theory** **22–01–2018**

1. (4 points) Evaluate $2^{2527} \% 77$ using Euler's theorem.

2. (4 points) Let a be a primitive root mod 38. Evaluate $|a^8|_{38}$.

3. (7 points) Solve the discrete logarithm problem $19^x \equiv 21 \pmod{22}$.

4. (8 points) Solve the quadratic congruence $x^2 \equiv 130 \pmod{133}$. Note: 133 is composite.

5. (7 points) Evaluate the Legendre symbol $\left(\frac{285}{311}\right)$. Note: 311 is prime.

6. (5 points) Let $\gcd(x, y) = 1$. Prove that if $x \mid k$ and $y \mid k$, then $xy \mid k$.

7. (5 points) Choose only one problem, (a) OR (b):

   (a) Let $m \equiv -n \pmod{17}$. Prove that if m is a primitive root mod 17, then n is a primitive root mod 17.

   (b) Let p be a prime number. Prove that if $p \equiv \pm 1 \pmod{12}$, then $\left(\frac{3}{p}\right) = +1$.

–Amin Witno