Philadelphia University
Department of Basic Sciences

Exam 2 **Number Theory** **04–01–2018**

1. (2 points) Count how many primitive roots exist mod 463. (Note: 463 is prime.)
2. (4 points) Solve the congruence $x^7 \equiv 3 \pmod{55}$.
3. (4 points) Solve the discrete logarithm problem $9^x \equiv 3 \pmod{13}$ with the help of the primitive root $g = 2$.
4. (3 points) Let a prime $p > 2$ and let k be a primitive root mod p. Prove that $k^{(p-1)/2} \equiv -1 \pmod{p}$.
5. (4 points) Use the Chinese remainder theorem and Fermat's little theorem to prove that $n^{61} \equiv n \pmod{143}$ for all integers n. (Note: 143 is composite.)
6. (3 points) Let c be an integer such that $c^8 \equiv -1 \pmod{17}$. Prove that c is a primitive root mod 17.

–Amin Witno