Philadelphia University

Department of Basic Sciences

Final Exam **Number Theory** **16–06–2015**

Each problem is worth 5 points; Write complete solution.

1. Use Fermat factorization to factor the number 5141.
2. Use Wilson's theorem to help compute $235\,!\,\%\,239$.
3. Find all the integer solutions for the system of three congruences:

$$\begin{aligned} x &\equiv 2 \pmod 4 \\ x &\equiv 3 \pmod 5 \\ x &\equiv 6 \pmod 7 \end{aligned}$$

4. Use Euler's theorem to help compute $5^{30500}\,\%\,49$.
5. Find all the integer solutions for the congruence $x^{23} \equiv 3 \pmod{55}$.
6. Count how many primitive roots exist mod 6250.
7. Use the primitive root $g = 3 \pmod{17}$ to solve the discrete logarithm problem $5^x \equiv 16 \pmod{17}$.
8. Find all the integer solutions for the quadratic congruence $x^2 \equiv 59 \pmod{85}$.

–Amin Witno

The list of prime numbers $p < 200$:

2	3	5	7	11	13	17	19	23	29
31	37	41	43	47	53	59	61	67	71
73	79	83	89	97	101	103	107	109	113
127	131	137	139	149	151	157	163	167	173
179	181	191	193	197	199				