Philadelphia University

Department of Basic Sciences

Final Exam **Number Theory** **02–06–2013**

Write complete solutions.

1. Suppose that $\gcd(m, n) = 1$ and both $m \mid k$ and $n \mid k$. Prove that $mn \mid k$.
2. Use Euler's theorem to evaluate $7^{56789} \,\%\, 60$.
3. Use Wilson's theorem to evaluate $100! \,\%\, 103$.
4. Use Chinese remainder theorem to find all integers x solution to the system of congruences.

$$\begin{aligned} x &\equiv 1 \pmod 3 \\ x &\equiv 2 \pmod 5 \\ x &\equiv 2 \pmod 8 \end{aligned}$$

5. Use the primitive root $g = 2$ modulo 11 to find all integers x solution to the discrete logarithm problem.

$$9^x \equiv 3 \pmod{11}$$

6. Evaluate the Legendre symbol $\left(\dfrac{154}{199}\right)$.
7. Let g be a primitive root modulo a prime $p > 2$. Prove that $\left(\dfrac{g}{p}\right) = -1$.
8. Use Chinese remainder theorem to find all integers x solution to the quadratic congruence.

$$x^2 \equiv 26 \pmod{55}$$

–Amin Witno

The list of prime numbers $p < 200$:

2	3	5	7	11	13	17	19	23	29
31	37	41	43	47	53	59	61	67	71
73	79	83	89	97	101	103	107	109	113
127	131	137	139	149	151	157	163	167	173
179	181	191	193	197	199				