

PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

Final Exam

Number Theory

22-01-2012

Solutions must be complete in order to receive full credit.

1. Let p be a prime number. Prove that if $3 \mid p - 1$ then $p \in [1]_6$.
2. Evaluate $310! \pmod{313}$ with the help of Wilson's theorem. Note that 313 is prime.
3. Evaluate $7^{2596} \pmod{405}$ with the help of Euler's theorem.
4. Find all solutions to the discrete logarithm problem $5^x \equiv 4 \pmod{11}$ using the primitive root $g = 2$.
5. Evaluate the Legendre symbol $\left(\frac{7250}{8111}\right)$.
6. Find all solutions to the congruence $x^2 \equiv 34 \pmod{55}$. Note that 55 is composite.
7. Let g be a primitive root modulo a prime number p . Prove that $\left(\frac{g}{p}\right) = -1$.

-Amin Witno