## PHILADELPHIA UNIVERSITY <br> DEPARTMENT OF BASIC SCIENCES

## Final Exam <br> MATHEMATICS FOR COMPUTING

Each problem, except the first, is 7 marks.

1. Write down the formula for the Taylor series of $f(x)$ with center $x=a$. (1 point)
2. Find the interval of convergence of $\sum_{n=1}^{\infty} \frac{(5-x)^{n}}{n^{4}}$
3. Find the reduced row echelon form of $A=\left[\begin{array}{ccccc}3 & 2 & 3 & -2 & 1 \\ 1 & 1 & 1 & 0 & 3 \\ 1 & 2 & 1 & -1 & 2\end{array}\right]$
4. Solve using Cramer's Rule: $\left[\begin{array}{ccc}1 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 1 & 4\end{array}\right]\left[\begin{array}{l}x \\ y \\ z\end{array}\right]=\left[\begin{array}{l}1 \\ 3 \\ 1\end{array}\right]$
5. Evaluate the determinant of $A=\left[\begin{array}{cccc}2 & 0 & 0 & 1 \\ 0 & 1 & 3 & -3 \\ -2 & -3 & -5 & 2 \\ 4 & -4 & 4 & -6\end{array}\right]$
6. Find the eigenvalues and eigenvectors of $A=\left[\begin{array}{ll}1 & 2 \\ 3 & 2\end{array}\right]$
7. The eigenvalues of $A=\left[\begin{array}{lll}3 & 1 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 4\end{array}\right]$ are $\lambda=3$ and $\lambda=4$. Find the eigenvectors.
8. The eigenvalues of $A=\left[\begin{array}{ll}1 & 1 \\ 4 & 1\end{array}\right]$ are $\lambda=3$ and $\lambda=-1$ with eigenvectors $\left[\begin{array}{l}1 \\ 2\end{array}\right]$ and $\left[\begin{array}{c}1 \\ -2\end{array}\right]$. Diagonalize the matrix $A$ and then use it to compute $A^{10}$.
