

PHILADELPHIA UNIVERSITY DEPARTMENT OF BASIC SCIENCES

Final Exam MATHEMATICS FOR COMPUTING 1–2–2006

Each problem, except the first, is 7 marks.

1. Write down the formula for the Taylor series of f(x) with center x = a. (1 point)

2. Find the interval of convergence of
$$\sum_{n=1}^{\infty} \frac{(5-x)^n}{n^4}$$
3. Find the reduced row echelon form of
$$A = \begin{bmatrix} 3 & 2 & 3 & -2 & 1 \\ 1 & 1 & 1 & 0 & 3 \\ 1 & 2 & 1 & -1 & 2 \end{bmatrix}$$

4. Solve using Cramer's Rule:
$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & 0 & 1 \\ 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

5. Evaluate the determinant of
$$A = \begin{bmatrix} 2 & 0 & 0 & 1 \\ 0 & 1 & 3 & -3 \\ -2 & -3 & -5 & 2 \\ 4 & -4 & 4 & -6 \end{bmatrix}$$

6. Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$

7. The eigenvalues of $A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 4 \end{bmatrix}$ are $\lambda = 3$ and $\lambda = 4$. Find the eigenvectors.

8. The eigenvalues of
$$A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$$
 are $\lambda = 3$ and $\lambda = -1$ with eigenvectors $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$. Diagonalize the matrix A and then use it to compute A¹⁰