Department of Basic Sciences — Philadelphia University

Mid Exam **Linear Algebra 2** **15–12–2021**

1. (3 points) Given the vectors $\{1 - x + 2x^2, 3x - x^2, 2 + x - x^2, 1 + 2x^2\}$
   (a) Do they span or not span P_2?
   (b) Are they linearly dependent (LD) or independent (LI)?
   (c) Are they a basis for P_2?

2. (4 points) Find $P_{C\to A}$ given $P_{A\to B} = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ and $P_{B\to C} = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$

3. (4 points) Given $[v]_B = (2, 0, -1)$, find $[v]_C$ using the transition matrix

$$P_{A\to C} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -1 \\ 4 & 1 & 1 \end{bmatrix} \quad \text{and} \quad P_{B\to A} = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -1 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

4. (4 points) Let $T(x, y, z) = (3x - y + z, 2x + 2y, x - y - z)$ and $S(x, y, z) = (x + y + 2z, 2x + y + 3z, y - z)$. Find the transformation $T(S(x, y, z))$ using matrix multiplication.

5. (3 points) Let $T(x, y) = (3x - 4y, x + 2y)$. Find the inverse transformation $T^{-1}(x, y)$.

6. (4 points) Given two basis for R^3: $B = \{(1, 0, 1), (0, 1, 1), (1, 1, 1)\}$ and $C = \{(2, 3, 4), (-1, -1, 2), (3, 2, 0)\}$, find $P_{C\to B}$

7. (4 points) Let $S = \{cx^2 + cx^4 \in P_4 \mid c \in \mathbb{R}\}$. Prove S is a subspace or not a subspace of P_4

8. (4 points) Find the nullity and a basis for the Null Space of $A = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$

–Amin Witno