## Department of Basic Sciences - Philadelphia University

## Mid Exam

## Graph Theory

28-11-2021
There are (5) pages and (16) questions.

1. (1 point) If $G$ is an 8 -regular graph with 64 edges, then find $\left|V_{G}\right|$.
2. (1 point) If the graph $G$ has degree sequence (5, 4, 4, 3, 3, 2, 2, 1), then find $\left|E_{G}\right|$.
3. (1 point) Find all the values of $n$ such that $C_{n} \subseteq K_{3,3}$.
4. (1 point) If degree sequence of $G$ is $(9,8,8,6,5,5,4,3,3,1)$, find the degree sequence of $\bar{G}$.
5. (1 point) If the graph $G$ is self-complementary with 16 vertices, then find $\left|E_{G}\right|$.
6. (1 point) Given the incidence matrix $Z$ of a graph, find the adjacency matrix $A$.

$$
Z=\left[\begin{array}{lllll}
1 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0
\end{array}\right]
$$

7. (1 point) Given 2 isomorphic graphs below with adjacency matrices $A$ and $B$. Find a permutation matrix $P$ (without proof) such that $P A P^{T}=B$.

8. (1 point) Find $\operatorname{deg}\left(\overline{P_{8}}\right)$.
9. (2 points) Apply the Degree Sequence Algorithm for the sequence (6, 5, 4, 4, 3, 2, 2, 1, 1) and determine graphical or not graphical.
10. (2 points) Draw 2 examples of a tree with degree sequence $(4,2,2,1,1,1,1)$ such that the first example is not isomorphic to the second example (without proof).
11. (3 points) The weight matrix $W$ is given. (a) Draw the graph then (b) Find the total weight of the Minimal Spanning Tree.
$W=\left[\begin{array}{cccccc}0 & 8 & 0 & 7 & 3 & 0 \\ 8 & 0 & 10 & 6 & 0 & 9 \\ 0 & 10 & 0 & 5 & 4 & 0 \\ 7 & 6 & 5 & 0 & 0 & 0 \\ 3 & 0 & 4 & 0 & 0 & 10 \\ 0 & 9 & 0 & 0 & 10 & 0\end{array}\right]$
12. (3 points) (a) Draw the Minimal Spanning Tree using Prim algorithm starting at vertex $v_{1}$ then (b) Find the MST sequence.

13. (2 points) Prove: If $G$ is a disconnected graph with 8 vertices and 15 edges, then $\bar{G}$ is cyclic.
14. (3 points) Draw the BFS spanning trees starting at vertex 1 , and find the BFS sequence.

15. (3 points) Repeat Problem (14) using DFS, starting at vertex 1.
16. (4 points) Use cofactor to count the number of labeled spanning trees of $G$.

