Department of Basic Sciences — Philadelphia University

Final Exam **Graph Theory** **11–06–2017**

Part 1. (1 point each) Multiple choice: circle one answer.

1. All trees are bipartite graphs.
   (a) True (b) False

2. If a graph contains no triangles, then it must be bipartite.
   (a) True (b) False

3. The graph $K_{m,n}$ is a Hamilton graph if and only if it has a perfect matching.
   (a) True (b) False

4. Which one is the correct formula for $\chi(C_n)$?
   (a) $n(n-1)/2$ (b) $(2n+1)/n$ (c) $2 + (n \bmod 2)$ (d) none of these

5. If we use Theorem 5.26.1 (If G is planar, then $e \leq 3n - 6$), which graph can be proved not planar?
   (a) K_6 (b) $K_{3,4}$ (c) $K_{3,5}$ (d) all of these

6. Which graph is the dual graph of $K_{2,n}$?
   (a) C_n (b) P_{n-1} (c) K_{n-1} (d) none of these

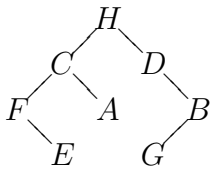
Part 2. (2 points each) Short answer: show your work.

7. Find all values of n such that $C_n \subseteq K_{3,5}$.
8. The degree sequence of G is $(5, 4, 3, 3, 2, 1, 1)$. Find $\deg(\overline{G})$.
9. A complete bipartite graph has 72 edges. Find the minimum number of vertices.
10. Draw 2 different examples of a self-complimentary graph with 5 vertices.

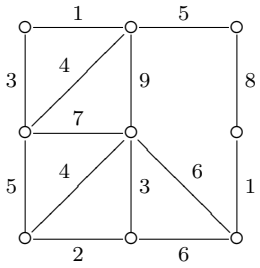
11. Given the incidence matrix Z, find the diameter of the graph.

$$Z = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

12. Given the binary tree, write the output using the in-order traversal algorithm.

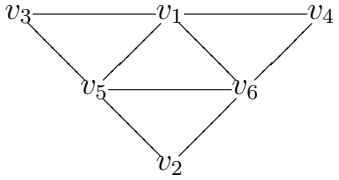


13. Draw the minimum spanning tree and compute its weight.

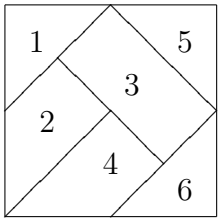


14. A plane graph with 10 vertices has degree 32. Find the number of regions.

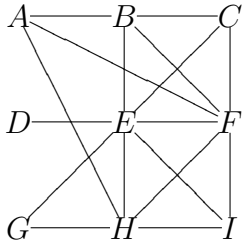
15. Color the graph using the Sequential Coloring Algorithm and also find the chromatic number.



16. Find the chromatic number of the map by coloring the dual graph.



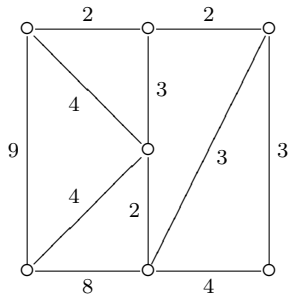
17. Color the graph using Welsh-Powell Algorithm.



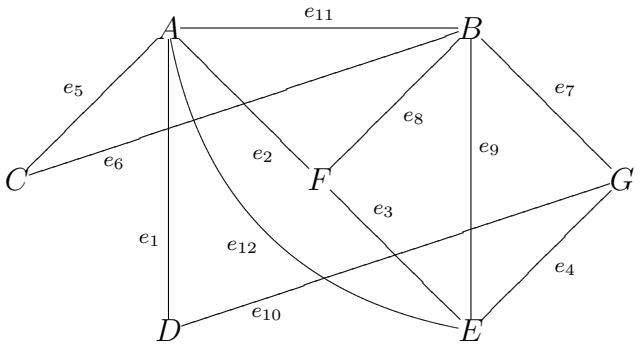
Part 3. (4 points each) Write complete solution.

18. Let G be a planar graph with at least 11 vertices. Prove that $\overline{G}$ is not planar.

19. Solve the Traveling Salesman Problem by drawing the 3 possible Hamilton cycles.



20. Use Hamilton cycle to determine if the graph is planar or not planar.



–Amin Witno