

Exam 1

Graph Theory

02-04-2017

Part I. (1 point each) Write short answer.

1. The degree sequence of G is $(4, 3, 2, 1, 1, 1)$. Find the degree sequence of \overline{G} .

2. The degree sequence $(5, 4, 4, 3, 3, 2, 1)$ is graphical. True or false?

(A) True

(B) False

3. Given the incidence matrix Z of a graph, find the adjacency matrix A .

$$Z = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

4. Which degree sequence comes from a tree? (Circle one answer.)

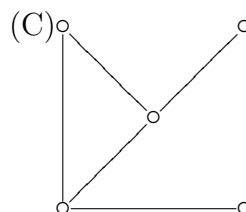
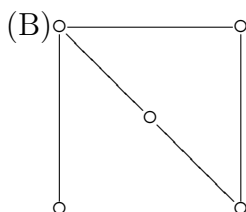
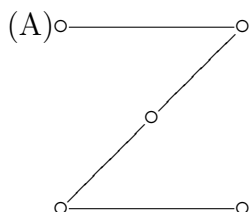
(A) 4,4,3,2,1,1,1

(B) 4,3,2,2,1,1,1

(C) 4,3,1,1,1,1,1

(D) none of these

5. Which graph is self-complementary? (Circle one answer.)



(D) none of these

6. Which graph contains C_5 ? (Circle one answer.)

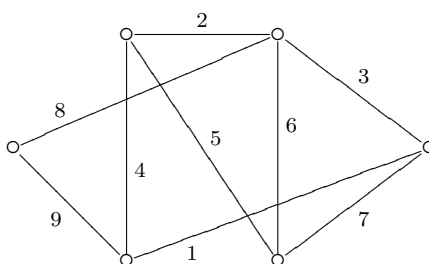
(A) C_6

(B) $K_{2,3}$

(C) P_6

(D) none of these

7. Find the the minimal spanning tree (MST) and its value.



8. Which graph G is connected and \overline{G} is also connected? (Circle one answer.)

- (A) P_3 (B) K_5 (C) $K_{3,3}$ (D) none of these

9. A tree with more than 1 vertex must have at least 2 leaves. True or false?

- (A) True (B) False

10. A connected regular graph has 17 edges. Find the number of vertices.

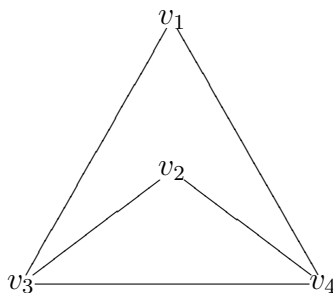
11. Which graph is cyclic and contains a bridge? (Circle one answer.)

- (A) $K_{1,5}$ (B) $K_{3,3}$ (C) C_6 (D) none of these

Part II. (3 points each) Write complete solution.

12. Draw 4 non-isomorphic trees with 6 vertices.

13. Find the number of spanning trees for the given labeled graph.



14. Let G be a tree. Prove that if G is self-complementary, then $G \approx P_4$.