Philadelphia University

Department of Basic Sciences

Final Exam **Graph Theory** **11–6–2006**

Part One (16 points)

1. My mobile phone is (a) on (b) off (c) silent (d) does not exist

2. Find the dual graph of $K_{2,29}$.
   (a) P_{28} (b) C_{29} (c) K_{28} (d) $K_{29,2}$

3. Which graph has the largest diameter?
   (a) $K_{43,68}$ (b) C_{13} (c) P_9 (d) K_{25}

4. Which degree sequence comes from a tree?
   (a) 3, 3, 3, 2, 2, 1 (b) 3, 3, 3, 2, 2, 1, 1, 1
   (c) 3, 3, 3, 2, 2, 1, 1 (d) 3, 3, 3, 2, 2, 1, 1, 1, 1, 1

5. The Chinese Postman Problem for the graph K_6 has best solution of length
   (a) 33 (b) 15 (c) 18 (d) 21

6. Which graph is not bipartite?
   (a) K_{10} (b) C_{10} (c) P_{10} (d) $K_{10,10}$

7. Which graph is homeomorphic to C_4?
   (a) C_5 (b) P_4 (c) K_4 (d) $K_{2,2}$

8. A plane graph is 3-regular with 12 vertices. What is the number of regions?
   (a) 32 (b) 4 (c) 26 (d) 8

Part Two (13 points)

1. What is the definition?

   (a) Euler walk

   (b) Hamilton cycle

   (c) Tournament

2. Draw an example of a self-complementary graph with 5 vertices.

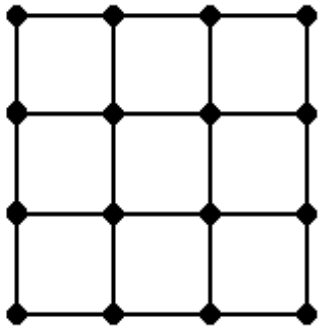
3. Draw an example of a connected graph with 4 vertices and no Euler walk.

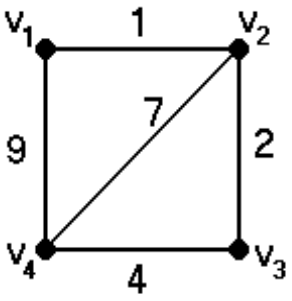
4. Draw an example of a plane graph with chromatic number equals 4.

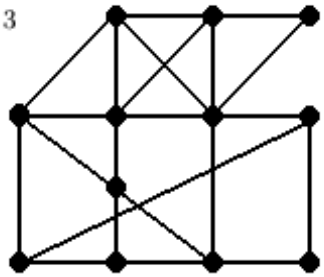
5. Draw an example of an anti-symmetric strongly connected digraph with 4 vertices.

6. Draw an example of a graph with 5 vertices and diameter equals 3.

Part Three (15 points)

G_1 

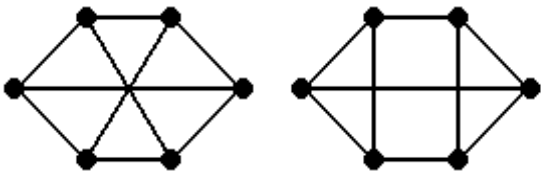
G_2 

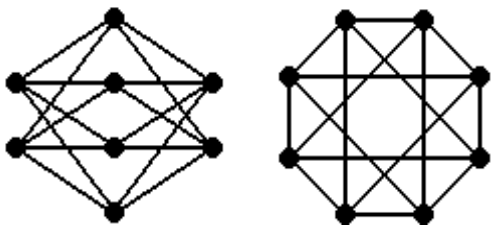
G_3 

1. (For the graph G_1)
   (a) Find and draw a spanning tree of G_1 using Depth-First Search Algorithm. Start at any vertex you like.
   (b) Use this spanning tree to turn G_1 into a strongly connected digraph by applying the One-Way Street Algorithm.
2. (For the graph G_2) Illustrate Floyd-Warshall Algorithm step-by-step (from $k = 1$ to $k = 4$) to find the distance matrix for G_2.
3. (For the graph G_3)
   (a) Apply the Maximum Color-Degree Algorithm to color the vertices of G_3. Label the vertices $v_1, v_2, \ldots, v_{12}$ according to the order of coloring.
   (b) Find $\chi(G_3)$.

Part Four (6 points)

1. Use Euler's Theorem to prove that a tree has at least two leaves.
2. For each pair of graphs, prove that they are isomorphic or prove that they are not isomorphic.

(a) 

(b) 

–Amin Witno