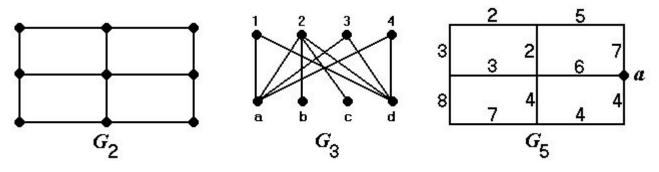
## PHILADELPHIA UNIVERSITY DEPARTMENT OF BASIC SCIENCES

## Graph Theory [Exam 1] 4–4–2006

Each problem is worth 4 points.

- 1. Definitions
  - (a) subgraph
  - (b) connected graph
  - (c) bipartite graph
  - (d) complete matching
- 2. Short Problems
  - (a) A complete graph G has 36 edges. Find  $|V_G|$ .
  - (b) A self-complementary graph G has 25 vertices. Find  $|E_G|$ .
  - (c) Draw a graph with degree sequence 4, 4, 4, 3, 3, 2, if possible.
  - (d) Redraw the graph  $G_2$  (see drawing below) as a bipartite graph, if possible.
- 3. Short Proofs
  - (a) Show why  $K_4$  and  $K_{2,2}$  are not isomorphic.
  - (b) Show why there is no tree with degree sequence 5, 4, 3, 2, 1, 1.
  - (c) Show why  $K_6$  is not bipartite.
  - (d) Use Hall's Marriage Theorem to show why the graph  $G_3$  (see drawing below) cannot have a perfect matching.
- 4. Count the number of labelled spanning trees of  $K_{2,3}$  using matrix cofactor.
- 5. Draw 2 minimum spanning trees for  $G_5$  (see drawing below), one using Kruskal's Algorithm and another by Prim's Algorithm (beginning at vertex a) showing step-by-step results.



-Amin Witno-