Graph Theory [Exam 1] 4–4–2006

Each problem is worth 4 points.

1. Definitions

    (a) subgraph

    (b) connected graph

    (c) bipartite graph

    (d) complete matching

2. Short Problems

    (a) A complete graph G has 36 edges. Find $|V_G|$.

    (b) A self-complementary graph G has 25 vertices. Find $|E_G|$.

    (c) Draw a graph with degree sequence $4, 4, 4, 3, 3, 2$, if possible.

    (d) Redraw the graph G_2 (see drawing below) as a bipartite graph, if possible.

3. Short Proofs

    (a) Show why K_4 and $K_{2,2}$ are not isomorphic.

    (b) Show why there is no tree with degree sequence $5, 4, 3, 2, 1, 1$.

    (c) Show why K_6 is not bipartite.

    (d) Use Hall's Marriage Theorem to show why the graph G_3 (see drawing below) cannot have a perfect matching.

4. Count the number of labelled spanning trees of $K_{2,3}$ using matrix cofactor.

5. Draw 2 minimum spanning trees for G_5 (see drawing below), one using Kruskal's Algorithm and another by Prim's Algorithm (beginning at vertex a) showing step-by-step results.

–Amin Witno–