Department of Basic Sciences — Philadelphia University

Exam 2 **Discrete Structures** **10–05–2017**

Part I. (8 questions, 1 point each) Circle one answer.

1. Let $|A| = 7$. Count how many subsets of A with at least 2 elements.

   (A) 120 (B) 247 (C) 502 (D) 1013

2. Count how many non-negative solutions in $A + B + C + D = 13$ with integers $A \geq 4$ and $B \geq 5$.

   (A) 10 (B) 20 (C) 35 (D) 56

3. Find the function $S(n)$ which gives the sequence 3, 4, 5, 3, 4, 5, 3, 4, 5, ...

   (A) $S(n) = 2 + n \bmod 2$ (B) $S(n) = 3 + n \bmod 2$

   (C) $S(n) = 2 + n \bmod 3$ (D) $S(n) = 3 + n \bmod 3$

4. Let $S(n) = 2\,S(n-1) + S(n-2)^2$ with $S(0) = 1$ and $S(1) = 1$. Find $S(4)$.

   (A) 14 (B) 23 (C) 32 (D) 40

5. Find the matrix corresponding to the relation $R = \{(x, y) \mid \lfloor \frac{x}{2} \rfloor = \lfloor \frac{y}{2} \rfloor\}$.

   (A) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

6. If $R = \{(1, 2), (2, 4), (3, 1), (4, 2)\}$ and $S = \{(1, 3), (2, 1), (3, 4), (4, 2)\}$, find $R \circ S$.

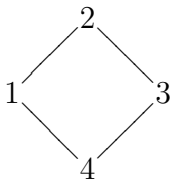
   (A) $\{(1, 1), (2, 2), (3, 3), (4, 1)\}$ (B) $\{(1, 1), (2, 2), (3, 2), (4, 4)\}$

   (C) $\{(1, 1), (2, 2), (3, 2), (4, 1)\}$ (D) $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$

7. Determine true or false for the relation $R = \{(1, 1), (1, 2), (2, 1), (3, 4)\}$:

   (A) symmetric (T) transitive (T) (B) symmetric (T) transitive (F)

   (C) symmetric (F) transitive (T) (D) symmetric (F) transitive (F)

8. Change the Hasse diagram 

to matrix.

(A)
$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

(B)
$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

(C)
$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

(D)
$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Part II. (3 questions, 4 points each) Write complete solution.

9. From 1 to 600, count how many integers are multiples of 16 or 20 or 28.

10. Find the formula for the function $S(n)$ given by the recurrence relation:

$$\begin{cases} S(n) = 2\,S(n-1) + 35\,S(n-2) \\ S(0) = 3 \\ S(1) = 7 \end{cases}$$

11. Given the relation matrix R, find the matrix for the transitive closure $\overline{R}$.

$$R = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

–Amin Witno