Department of Basic Sciences — Philadelphia University

Exam 2 **Discrete Structures** **23–12–2015**

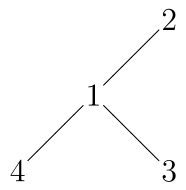
Part I. (2 points each) Circle one answer from the multiple choice.

1. The sequence 2, 6, 10, 14, 18, 22, ... is given by the function $S_n =$
   (A) $n^2 + 2$ (B) $4n + 2$ (C) $2^n + 1$ (D) $2n + 2$

2. If $R = \{(1,2),(1,3),(3,1)\}$ then $R^{-2} =$
   (A) $\{(1,1),(2,3),(3,3)\}$ (B) $\{(1,1),(3,2),(3,3)\}$
   (C) $\{(1,1),(2,1),(3,3)\}$ (D) $\{(1,1),(1,2),(3,3)\}$

3. The matrix $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ represents a relation that is
   (A) reflexive (F); symmetric (F); anti-symmetric (F); transitive (T)
   (B) reflexive (F); symmetric (F); anti-symmetric (F); transitive (F)
   (C) reflexive (F); symmetric (F); anti-symmetric (T); transitive (T)
   (D) reflexive (F); symmetric (F); anti-symmetric (T); transitive (F)

4. Let $A = \{1,2,3,4,5\}$. Which relation on A is a partial order?
   (A) $R = \{(a,b) \mid a \bmod b = 0\}$ (B) $R = \{(a,b) \mid a \bmod 2 = b \bmod 2\}$
   (C) $R = \{(a,b) \mid a \bmod b = 1\}$ (D) $R = \{(a,b) \mid a \bmod 2 = b \bmod 3\}$

5. Convert the Hasse diagram  to matrix.

   (A) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

Part II. (5 points each) Write complete solutions.

6. Find the function S_n given the following recurrence $S_n = S_{n-1} + 20S_{n-2}$ with $S_0 = 1$ and $S_1 = 2$.

7. Use induction to prove the following formula for all integers $n \geq 1$.

$$1 + 8 + 64 + \cdots + 8^n = \frac{8^{n+1} - 1}{7}$$

–Amin Witno