



**PHILADELPHIA UNIVERSITY**  
**DEPARTMENT OF BASIC SCIENCES**

**Final Exam A**

**DISCRETE STRUCTURES**

**01-06-2013**

**PART (I)** Each problem is worth 2 points. Circle one answer.

1) Convert the proposition  $p \leftrightarrow q$  to DNF.

- a)  $(p \wedge \neg q) \vee (\neg p \wedge q)$       b)  $(p \wedge q) \vee (\neg p \wedge q)$   
c)  $(p \wedge \neg q) \vee (\neg p \wedge \neg q)$       d)  $(p \wedge q) \vee (\neg p \wedge \neg q)$

2) Let  $A = \{1, 2, 3, 4, 5\}$  and  $B = \{1, 3, 5, 7\}$  and  $C = \{2, 4, 6\}$ . Then  $(A \oplus B) - C =$

- a)  $\{6\}$       b)  $\{7\}$       c)  $B$       d)  $C$

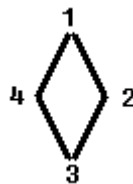
3) Let  $A = \{1, 2, 3, 4\}$ . Which relation  $R$  is symmetric on  $A$  ?

- a)  $R = \{(a,b) \mid a - b < 0\}$       b)  $R = \{(a,b) \mid a < b + 2\}$   
c)  $R = \{(a,b) \mid a + b < 5\}$       d)  $R = \{(a,b) \mid b < a + 2\}$

4) Let  $R = \{(1,2), (2,3), (3,3), (4,2)\}$ . Then  $R^2 =$

- a)  $\{(1,3), (2,3), (3,4)\}$       b)  $\{(1,3), (2,3), (3,3), (4,2)\}$   
c)  $\{(1,3), (2,3), (4,3)\}$       d)  $\{(1,3), (2,3), (3,3), (4,3)\}$

5) Which matrix corresponds to the given Hasse diagram?



- a)  $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$       b)  $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$       c)  $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$       d)  $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

6) Which graph has the largest degree?

- a)  $C_{10}$       b)  $P_{10}$       c)  $K_{10}$       d)  $K_{5,5}$

7) Which graph has diameter 2?

- a)  $C_6$       b)  $C_5$       c)  $P_5$       d)  $K_4$

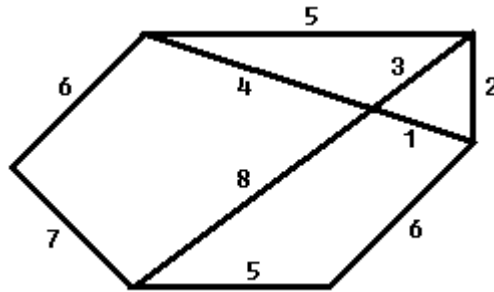
8) Convert the incidence matrix  $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$  to adjacency matrix.

- a)  $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$     b)  $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$     c)  $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$     d)  $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

9) Which graph has an Euler circuit?

- a)  $K_{2,9}$     b)  $P_{10}$     c)  $K_{10}$     d)  $K_{10,10}$

10) Find the weight of the minimal spanning tree (MST) for the given graph.



- a) 21    b) 23    c) 24    d) 28

**PART (II)** Each problem is worth 5 points. Write complete solutions.

11) Evaluate  $\text{GCD}(9999, 444)$ .

12) How many permutations with  $\{A, B, C, D, E, F, G, H\}$  contain "BACH" or "FED" ?

13) Find the formula for the sequence  $S(n)$  given by the recurrence relation.

$$\begin{aligned} S(n) &= 4 S(n-1) - 4 S(n-2) \\ S(0) &= 1 \\ S(1) &= 6 \end{aligned}$$

14) Solve the Chinese postman problem (CPP) for the given graph.

