



**PHILADELPHIA UNIVERSITY**  
**DEPARTMENT OF BASIC SCIENCES**

**Second Exam A**

**DISCRETE STRUCTURES**

**30-04-2013**

Part 1 Each problem is worth 2 points. Circle one answer.

1) The sequence 3, 7, 11, 15, 19, 23, ... is given by the function

a)  $S(n) = 4n + 3$

b)  $S(n) = (n+2)^2 + 3$

c)  $S(n) = 3n + 4$

d)  $S(n) = (n+1)^2 + 4$

2) Let  $A = \{1, 2, 3, 4\}$  and  $R = \{(a,b) \mid a - b < 1\}$ . Which one is correct?

a) Reflexive (F); Symmetric (T); Anti-symmetric (F); Transitive (F)

b) Reflexive (F); Symmetric (T); Anti-symmetric (F); Transitive (T)

c) Reflexive (T); Symmetric (F); Anti-symmetric (T); Transitive (T)

d) Reflexive (T); Symmetric (F); Anti-symmetric (T); Transitive (F)

3) Let  $R = \{(1,2), (2,3), (3,2)\}$ . Find the transitive closure of R.

a)  $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$  b)  $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$  c)  $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$  d)  $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

4) Find the equivalence classes from the equivalence relation

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

a)  $\{1, 3\}, \{2, 4\}$

b)  $\{1, 2\}, \{3, 4\}$

c)  $\{1, 4\}, \{2, 3\}$

d)  $\{1, 2\}, \{3\}, \{4\}$

Part 2 Each problem is worth 4 points. Write complete solution.

5) Find the formula for the recursive sequence  $S(n) = -2 S(n-1) + 15 S(n-2)$  given that  $S(0) = 1$  and  $S(1) = 1$ .

6) Prove the formula for all integers  $n \geq 1$  using induction.

$$1 + 5 + 25 + \dots + 5^{n-1} = \frac{5^n - 1}{4}$$

7) Let  $A = \{2, 8, 4, 6, 12\}$  and  $R = \{(a,b) \mid b \bmod a = 0\}$ .

a) Draw the digraph of R.

b) Prove that R is a partial order relation.

c) Draw the Hasse diagram.

--Amin Witno  
--Ameina Al-Taani