## PHILADELPHIA UNIVERSITY DEPARTMENT OF BASIC SCIENCES

## Second Exam A

Part 1 Each problem is worth 2 points. Circle one answer.

1) Given $R=\{(1,3),(2,1),(3,4),(4,2)\}$. Find $R^{-3}$
a) $\{(1,4),(2,3),(3,2),(4,1)\}$
b) $\{(1,3),(2,1),(3,4),(4,2)\}$
c) $\{(1,2),(2,1),(3,4),(4,3)\}$
d) $\{(1,3),(2,4),(3,2),(4,1)\}$
2) Given $A=\{1,2,3,4\}$ and $R=\{(a, b) \mid a+b<7\}$. Which one is correct?
a) reflexive (T); symmetric (F); anti-symmetric (T); transitive (F)
b) reflexive (T); symmetric (F); anti-symmetric (T); transitive (T)
c) reflexive ( F ); symmetric ( T ); anti-symmetric ( F ); transitive ( F )
d) reflexive ( F ); symmetric ( T ); anti-symmetric ( F ); transitive ( T )
3) Given $R=\{(1,1),(1,2),(1,4),(2,1),(2,2),(2,4),(3,3),(4,1),(4,2),(4,4)\}$.

Find the equivalence classes.
a) $\{1,3\},\{2\},\{4\}$
b) $\{1,2,4\},\{3\}$
c) $\{1,3\},\{2,4\}$
d) $\{1,3,4\}$, $\{2\}$
4) Which relation is a total order?
a) $\left[\begin{array}{lll}1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1\end{array}\right]$
b) $\left[\begin{array}{lll}1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1\end{array}\right]$
c) $\left[\begin{array}{lll}1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1\end{array}\right]$
d) $\left[\begin{array}{lll}1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1\end{array}\right]$
5) What is the transitive closure of $R=\{(1,2),(2,3),(3,1)\}$ ?
a) $\left[\begin{array}{lll}1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1\end{array}\right]$
b) $\left[\begin{array}{lll}0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0\end{array}\right]$
c) $\left[\begin{array}{lll}1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1\end{array}\right]$
d) $\left[\begin{array}{lll}1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1\end{array}\right]$
6) Given $A=\{1,2,3\}$ and $B=\{2,3,4\}$. How many relations from $A$ to $B$ ?
a) 512
b) 256
c) 128
d) 64

Part 2 Each problem is worth 4 points. Write complete solution.
7) Give example of a relation $R$ on $A=\{1,2,3,4\}$, one for (a) and one for (b).
(a) reflexive (T); symmetric (T); anti-symmetric (F); transitive (F)
(b) reflexive (F); symmetric (T); anti-symmetric (F); transitive (T)
8) Let $A=\{1,2,5,10,20\}$ and $R=\{(a, b) \mid b \bmod a=0\}$
a) Find the elements of R.
b) Draw the digraph.
c) Prove that $R$ is a partial order relation.
d) Draw the Hasse diagram.

