PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

Final Exam A **DISCRETE STRUCTURES** **21–01–2009**

PART (I) Each problem is worth 2½ points. Circle one answer.

1) Find a proposition equivalent to $\neg p \rightarrow (p \vee q)$.

   a) $p \rightarrow q$ b) $q \rightarrow p$ c) $p \rightarrow \neg q$ d) $\neg p \rightarrow q$

2) Evaluate GCD (2009, 9200).

   a) 1 b) 2 c) 7 d) other answer

3) How many positive integers ≤ 100 are multiples of 8 or 12 ?

   a) 12 b) 13 c) 15 d) 16

4) Let A = {1, 2, 3, 4} and R = {(a,b) | a + b > 1}. Then R is

   a) symmetric (T), transitive (T) b) symmetric (T), transitive (F)
   c) symmetric (F), transitive (T) d) symmetric (F), transitive (F)

5) Which equivalence relation has equivalence classes {1,2,5} and {3,4} ?

a $\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$ b $\begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$ c $\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$ d $\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$

6) Convert the incidence matrix $\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ to adjacency matrix.

   a) $\begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 1 \\ 2 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 0 & 2 & 1 \\ 0 & 1 & 0 & 1 \\ 2 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 0 & 2 & 1 \\ 0 & 1 & 1 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

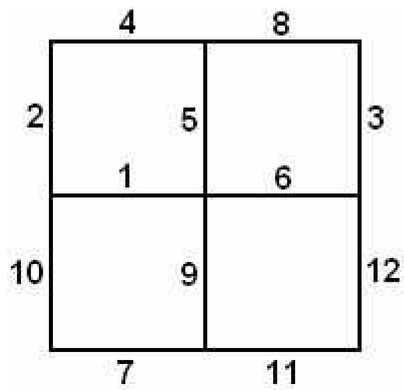
7) A complete graph has 91 edges. How many points does it have?

   a) 14 b) 15 c) 16 d) 17

8) Which graph is an Euler path (not Euler circuit)?

a) K5 b) K6 c) K5,2 d) K5,4

9) Find the minimal spanning tree. The total value is



a) 36 b) 39 c) 41 d) 43

10) Find the dual graph of K4.

a) K1 b) K2 c) K3 d) K1,2

PART (II) Each problem is worth 5 points. Write complete solutions.

11) Prove: If $x^2 - 10x + 3$ is odd then x is even.

12) Find an explicit formula for the recurrence relation given by

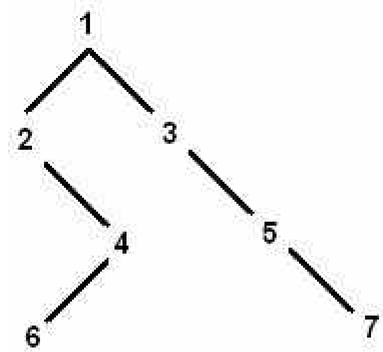
$f(0) = 1$
$f(1) = 2$
$f(n) = 2\,f(n-1) + 3\,f(n-2)$

13) Give one example of a relation R on A = {1,2,3,4} for each (a) and (b).

(a) reflexive (T); symmetric (T); anti-symmetric (F); transitive (F)
(b) reflexive (F); symmetric (T); anti-symmetric (F); transitive (T)

14) Let A = {2,3,6,8,24} and R = {(a,b) | b mod a = 0}. Draw the Hasse diagram.

15) Find the output using the algorithm (a) pre-order (b) post-order (c) in-order.



-Amin Witno