PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

Final Exam A **DISCRETE STRUCTURES** **04–02–2008**

PART (I) Each problem is worth 3 points. Circle one answer.

1) Which proposition is equivalent to $p \rightarrow q$?

   a) $\neg p \rightarrow \neg q$ b) $\neg p \vee q$
   c) $q \rightarrow p$ d) $\neg q \vee p$

2) The number 2008 is decimal. Convert it to hexadecimal.

   a) 7D8 b) 820 c) 728 d) 8D0

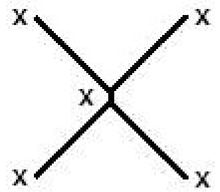
3) How many integer solutions of the equation $x + y + z = 20$ such that $x \geq 3$ and $y \geq 5$ and $z \geq 1$?

   a) 78 b) 66 c) 55 d) 45

4) Let $A = \{0, 1, 4, 6, 9\}$ and R is an equivalence relation on A given by $R = \{(a,b) \mid a \bmod 4 = b \bmod 4\}$. Find the equivalence classes.

   a) {0, 4, 6}, {1, 9} b) {0}, {1, 4}, {6, 9}
   c) {0}, {1, 6}, {4, 9} d) {0, 4}, {1, 9}, {6}

5) A partial order relation is given by this Hasse diagram. Find the zero-one matrix.



$$a \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix} \quad b \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix} \quad c \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad d \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

6) A complete graph K_n has 66 edges. How many points does it have?

   a) 14 b) 24 c) 12 d) 28

7) Which graph is an Euler path but not Euler circuit?

a) K 11　　b) K 2,10　　c) K 2,11　　d) K 10,11

8) Convert the incidence matrix $\begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$ to adjacency matrix.

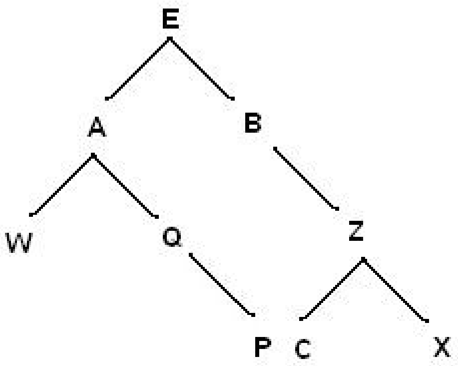
a) $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 0 & 2 \\ 0 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

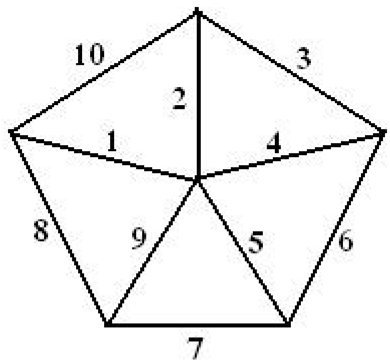
9) Find the output using the in-order algorithm. (The tree on the left)

a) W-P-Q-A-B-C-X-Z-E　　b) W-P-Q-A-C-X-Z-B-E
c) W-A-Q-P-E-C-B-Z-X　　d) W-A-Q-P-E-B-C-Z-X

10) Find the minimum spanning tree. (The graph on the right) The value is

a) 17　　b) 18　　c) 19　　d) 20





PART (II)　　Each problem is worth 4 points. Write complete solutions.

11) Convert the proposition $(p \leftrightarrow r) \rightarrow q$ to a CNF.

12) Prove: If $x^2 - 8x + 5$ is odd then x is even.

13) Find an explicit formula for the recurrence relation given by

$f(0) = 1$
$f(1) = 2$
$f(n) = -f(n-1) + 12\,f(n-2)$

14) How many positive integers ≤ 300 which are multiples of 6 or 8 or 9 ?

15) Let $A = \{1, 2, 3, 4\}$. Give an example of a relation on A which is

a) reflexive, symmetric, not transitive
b) symmetric, transitive, not reflexive

-Amin Witno