PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

Second Exam A **DISCRETE STRUCTURES** **26–12–2006**

Part 1 Each problem is worth 2 points. Circle one answer.

1) A sequence satisfies the recurrence relation $f(n) = 4\,f(n–1) – 4\,f(n–2)$ with $f(0) = 1$ and $f(1) = 4$. Find an explicit formula for $f(n)$.
a) $2^n + n\,(–2)^n$ b) $2^n + 2n\,(–2)^n$
c) $2^n + n\,(2^n)$ d) $2^n + 2n\,(2^n)$

2) Suppose $A \subseteq B$. Then $A \oplus B =$
a) $B – A$ b) $A – B$ c) $A \cap B$ d) $A \cup B$

3) There are 6 chapters in Discrete Mathematics. What is the minimum number of questions in the exam so that at least 13 come from the same chapter?
a) 67 b) 78 c) 73 d) 72

4) How many different permutations can be formed using the elements of the multiset {A, B, C, C, B, B, A}?
a) 840 b) 420 c) 105 d) 210

5) An equivalence relation is represented by the following zero-one matrix.

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Find the equivalence classes.
a) {1, 2, 3, 5}, {4} b) {1, 3}, {2, 5}, {4}
c) {1, 3, 4}, {2, 5} d) {1, 3, 4}, {2}, {5}

Part 2 This part is worth 10 points. Write complete solutions for full credit.

6) Let $A = \{1, 2, 3, 4\}$. Give an example of $R \subseteq A \times A$ with the given properties.
a) reflexive (F) symmetric (T) anti-symmetric (F) transitive (T)
b) reflexive (F) symmetric (F) anti-symmetric (F) transitive (F)

7) How many positive integers ≤ 500 which are not multiples of 6 or 9 or 15?

8) Let $A = \{2, 4, 8, 12, 24\}$ and $R = \{(a, b) \mid a \text{ divides } b\} \subseteq A \times A$.
a) Find the elements of R and draw the digraph.
b) Prove that R is a partial order relation and draw the Hasse diagram.

-Amin Witno