# PHILADELPHIA UNIVERSITY
## DEPARTMENT OF BASIC SCIENCES

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| Discrete Structures | (210104) | Paper: | Exam 1 Form (A) |
| Discrete Mathematics | (210242) | Date: | 5 April 2005 |
| Discrete Mathematics | (250151) | Time: | 15:00 – 15:50 |

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**PART 1** Circle the best answer. (2 points each)

1. Evaluate LCM (60, 39).
   (a) 180 (b) 1260 (c) 2340 (d) 780

2. Convert the decimal number 222 to binary.
   (a) 1011110 (b) 111101 (c) 11110 (d) 111110

3. Convert the binary number 1101101010 to hexadecimal.
   (a) DA8 (b) DA2 (c) 36A (d) C6A

4. Convert the proposition $(p \wedge \neg q) \vee (\neg p \wedge \neg q)$ to a CNF.
   (a) $(p \vee \neg q) \wedge (\neg p \vee q)$ (c) $(\neg p \vee \neg q) \wedge (p \vee \neg q)$
   (b) $(\neg p \vee q) \wedge (p \vee q)$ (d) $(p \vee q) \wedge (\neg p \vee \neg q)$

5. Let $P(x, y) : x^2 \geq y^2$. One of these propositions is false.
   (a) $\forall x \exists y\ P(x, y)$ (c) $\forall y \exists x\ P(x, y)$
   (b) $\exists x \forall y\ P(x, y)$ (d) $\exists y \forall x\ P(x, y)$

6. One of these propositions is equivalent to $C \rightarrow (B \rightarrow A)$.
   (a) $(C \rightarrow B) \rightarrow A$ (c) $B \rightarrow (C \rightarrow A)$
   (b) $(B \rightarrow C) \rightarrow A$ (d) $A \rightarrow (C \rightarrow B)$

**PART 2** Write complete solutions in the space provided. (4 points each)

1. Prove that if $X^2 + 4X$ is even then X is even.

2. Prove for all integers $n \geq 1$: $1+5+25+\ldots+5^{n-1}=\frac{5^n-1}{4}$

**ANSWERS** D, A, C, C, B, C

1)
p: $X^2 + 4X$ is even
q: X is even
We will prove $p \rightarrow q$ using $\neg q \rightarrow \neg p$

$\neg q$: X is odd
$\rightarrow$ $X = 2n + 1$ (n integer)
$\rightarrow$ $X^2 + 4X = (2n + 1)^2 + 4(2n + 1)$
$\rightarrow$ $X^2 + 4X = 4n^2 + 4n + 1 + 8n + 4$
$\rightarrow$ $X^2 + 4X = 4n^2 + 12n + 5$
$\rightarrow$ $X^2 + 4X = 2(2n^2 + 6n + 2) + 1$
$\rightarrow$ $X^2 + 4X = 2m + 1$ (m = is an integer)
$\rightarrow$ $X^2 + 4X$ is odd
$\rightarrow \neg p$

2) P(n): $1+5+25+\ldots+5^{n-1}=\frac{5^n-1}{4}$

We will prove $\forall n$ P(n) using induction

a) P(1): $1=\frac{5-1}{4}$ is true

b) P(x): $1+5+25+\ldots+5^{x-1}=\frac{5^x-1}{4}$

$\rightarrow \quad 1+5+25+\ldots+5^{x-1}+5^x=\frac{5^x-1}{4}+5^x$

$\rightarrow \quad 1+5+25+\ldots+5^x=\frac{5^x-1+4(5^x)}{4}$

$\rightarrow \quad 1+5+25+\ldots+5^x=\frac{5(5^x)-1}{4}$

$\rightarrow \quad 1+5+25+\ldots+5^x=\frac{5^{x+1}-1}{4}$

$\rightarrow$ P(x + 1)