Department of Basic Sciences — Philadelphia University

Mid Exam   **Complex Analysis**   **29–11–2021**

1. (2 points) Evaluate $\frac{3-6i}{-2+i}$ in the rectangular form $(X) + (Y)i$.

2. (2 points) Evaluate $\text{Log}(-\sqrt{3} - i)$ in the rectangular form $(X) + (Y)i$.

3. (2 points) Evaluate the Principal power $(-2)^i$ in the rectangular form $(X)+(Y)i$.

4. (3 points) Evaluate $\sinh(\frac{\pi}{4}i)$ in the rectangular form $(X) + (Y)i$.

5. (3 points) Evaluate $(-1+i)^{10}$ —Final answer in rectangular form $(X)+(Y)i$.

6. (4 points) Find all complex numbers z such that $\cosh z = 1$.

7. (2 points) Let $f(z) = \sin(z^2)$. Find the real functions $u(x,y)$ and $v(x,y)$ such that $f(z) = u + iv$.

8. (3 points) Let $f(z) = ie^{(1+i)z}$. Find the real functions $u(x,y)$ and $v(x,y)$ such that $f(z) = u + iv$.

9. (2 points) Use the definition of limit to prove $\lim_{z \to (1-i)} z + 5iz = 6 + 4i$

10. (2 points) Prove the limit at infinity: $\lim_{z \to \infty} \dfrac{iz^3 - 2z + 3i}{z^2 - i} = \infty$

11. (5 points) Let $f(z) = e^x(y^2 + iy^2 - 8i)$. Use Cauchy-Riemann equations to find the domain where $f'(z)$ exists, then find $f'(z)$.

–Amin Witno