PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

**Final Exam** **Complex Analysis** **29–05–2018**

1. (5 points) Find all the numbers $z = x + iy$ such that $z^2 = 21 - 20i$.

2. (7 points) Find the domain where $f'$ exists and find it using the Cauchy-Riemann equations.
$$f(z) = f(x + iy) = 2x^2 - y^3 + i(x + 2xy - y^2)$$

3. (4 points) Evaluate $(-1 - i\sqrt{3})^{2i}$ using the principal Log. Write your answer in the form $x + iy$, where $x, y \in \mathbb{R}$.

4. (7 points) Evaluate $\int_C (\bar{z})^2 \, dz$, where $C$ is the straight line from $-1+2i$ to $1-2i$. Write your answer in the form $x + iy$, where $x, y \in \mathbb{R}$.

5. (5 points) Use Cauchy Integral Formula to evaluate the contour integral, where $C$ is the circle $|z - 2i| = 3$.
$$\int_C \frac{dz}{z(z^2 + 4)}$$

6. (5 points) Evaluate the real integral using Cauchy Integral Formula.
$$\int_{-\infty}^{\infty} \frac{dx}{(x^2 + 4)^3}$$

7. (7 points) Evaluate the real integral using Cauchy Integral Formula.
$$\int_0^{2\pi} \frac{dx}{5 - 3\sin x}$$

–Amin Witno