Philadelphia University

Department of Basic Sciences

Exam 1 **Complex Analysis** **17–04–2018**

1. (3 points)
   (a) Simplify in the form $x + iy$:
   $$\frac{3 - 2i}{-1 + i}$$
   (b) Write the number $z = -1 + i\sqrt{3}$ in polar form $z = re^{i\theta}$, where $\theta = \text{Arg } z$.
   (c) Draw the region in the complex plane given by the condition $|2z + 3i| \leq 4$.

2. (3 points) Find two complex numbers $z = x + iy$ such that $z^2 = -15 - 8i$.

3. (2 points) Find the functions $u(x, y)$ and $v(x, y)$ such that $f(z) = u + iv$:
   $$f(z) = \bar{z} - ie^{i|z|}$$

4. (4 points) Use the definition of limit to prove the limit:
   $$\lim_{z \to 2+i} 3z - 2iz = 8 - i$$

5. (4 points) Let $f(z) = e^x(y^2 + iy - 3i)$.
   (a) Use Cauchy-Riemann equations to find the domain where $f'(z)$ exists.
   (b) Find $f'(z)$.

6. (4 points) Let $u(x, y) = xy + e^x \cos y$.
   (a) Prove that $u(x, y)$ is harmonic for all $x, y \in \mathbb{R}$.
   (b) Find a harmonic conjugate $v(x, y)$ such that $f(z) = u + iv$ is entire.

–Amin Witno