Philadelphia University
Department of Basic Sciences

Final Exam **Abstract Algebra 2** **13–06–2016**

1. (5 points) Let $\theta : R \to R'$ be a ring homomorphism. Prove that θ is one-to-one if and only if $\ker \theta = \{0\}$.
2. (5 points) Let F be a field. Prove that F has only two ideals: $I = \{0\}$ and $I = F$.
3. (5 points) Find the minimal polynomial for $\sqrt{13} - \sqrt{3}$ over $\mathbb{Q}$.
4. (5 points) Prove reducible or irreducible: $f = 3x^4 - 5x^3 + 6x^2 - 2x + 7 \in \mathbb{Q}[x]$.
5. (10 points) Let R be a commutative ring, and let $x, y \in R$. Prove that the set $S = \{ax + by \mid a, b \in R\}$ is an ideal of R.
6. (10 points) Let F be a finite field with $\chi(F) = p$. Let $\theta : F \to F$ such that $\theta(a) = a^{p^2}$. Prove that θ is an isomorphism.

–Amin Witno