Philadelphia University

Department of Basic Sciences

Exam 1 **Abstract Algebra 2** **25–03–2014**

Choose five problems. No bonus.

1. (a) Find an example of a ring with no unity.
   (b) Find an example of a finite ring with no zero divisor.
   (c) Find an example of an integral domain that is not a field.
   (d) Find an example of a unit element in $\mathbb{Z} \times \mathbb{Z}$ that is not the unity.
2. Let $S = \{\begin{pmatrix} a & b \\ -b & a \end{pmatrix} \mid a, b \in \mathbb{R}\}$. Prove that S is a subring of $M(2, \mathbb{R})$.
3. Let R be a ring with unity but without zero divisors. Prove that for any two elements $a, b \in R$, if $ab = 1$ then $ba = 1$.
4. Prove that if R is a finite integral domain, then R is a field.
5. Let $R = \mathbb{Z}_4 \times \mathbb{Z}_4$ and let $a = (2, 2) \in R$. Construct the multiplication table for the factor ring $R/(a)$.
6. Let $\theta : R \to R'$ be a ring homomorphism. Prove that $\ker(\theta)$ is an ideal of R.

–Amin Witno