Philadelphia University

Department of Basic Sciences

Exam 1 **Abstract Algebra 2** **14–03–2012**

Choose four problems.

1. (a) What is the definition of a ring? (b) Let G be an abelian group under addition. Suppose that $a \times b = 0$ for all $a, b \in G$. Prove that G is a ring.
2. Let R be a ring and let $S = \{a \in R \mid ar = ra \forall r \in R\}$. Prove that S is a subring of R.
3. Let $R = \mathbb{Z}_5 \times \mathbb{Z}_3$. (a) What is the definition of an integral domain? (b) Prove that R is not an integral domain. (c) What are the unit elements in R? (d) What are the zero divisors in R?
4. (a) What is the definition of a field? (b) Find an example of an integral domain which is not a field. (c) Let R be a finite integral domain. Prove that R is a field.
5. Let $S = \{a + b\sqrt{3} \mid a, b \in \mathbb{Q}\}$. Prove that S is a subfield of $\mathbb{R}$.
6. Let I be an ideal of a commutative ring R with unity. (a) What is the definition of an ideal? (b) Suppose that R has no ideals except $\{0\}$ and R. Prove that R is a field.

–Amin Witno