Philadelphia University

Department of Basic Sciences

Exam 1 **Abstract Algebra 2** **30–03–2011**

Choose 3 problems from the following 5 problems.

1. (a) Write the definition of a ring.
   (b) Let R be a ring with unity. Write the definition of unity.
   (c) Let $a, b \in R$ such that $ab = 1$. Prove that $ba = 1$.
2. (a) Write the definition of an integral domain.
   (b) Write the definition of a field.
   (c) Prove that a finite integral domain is a field.
3. (a) Write the definition of a principal ideal.
   (b) Write the definition of a principal ideal domain.
   (c) Is $\mathbb{Q}$ a principal ideal domain? Prove why or why not.
4. (a) Write the definition of an ideal I of a ring R.
   (b) Write the definition of the factor ring R/I.
   (c) Construct the Cayley tables (both addition and multiplication) for the factor ring $\mathbb{Z}_{12}/(8)$.
5. (a) Write the definition of a ring homomorphism.
   (b) Let $\theta : R \to R'$ be a ring homomorphism which is onto. If R has a unity, prove that $\theta(1) = 1$.
   (c) If θ is not onto, find an example where $\theta(1) \neq 1$.

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