PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

Exam 2 **Abstract Algebra 2** **26–04–2009**

Choose any 4 problems from the following 6 problems.

1. (a) What is the definition of a ring homomorphism? (b) Prove that $\theta(a+bi) = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is a one-to-one homomorphism from the complex ring $\mathbf{C}$ to the matrix ring $M(2, \mathbf{R})$.
2. Let $R[x]$ be a polynomial ring. (a) Prove that $S = \{f \in R[x] \mid \deg(f) = 0\}$ is a subring of $R[x]$. (b) Prove that S is not an ideal.
3. (a) What is the definition of a principal ideal domain? (b) Let F be a field. Prove that the ring $F[x]$ is a principal ideal domain.
4. (a) What is the definition of $\gcd(f, g)$ in $\mathbf{Q}[x]$? (b) Evaluate $\gcd(x^{96} - 1, x^{27} - 1)$.
5. (a) What is the definition of an irreducible polynomial? (b) Prove that the factor ring $F[x]/(f)$ is a field if and only if f is irreducible.
6. Let $f = x^2 - 2$. (a) Prove that f is irreducible in $\mathbf{Z}_{11}$. (b) Prove that f is reducible in $\mathbf{Z}_{17}$.

Notes:

1. Full credit will only be given to a solution which is logically correct. Be very careful in what you write!
2. You may assume all the theorems given in the notes, unless when the problem asks you to prove the theorem.
3. Do not spend too much time on a single problem. Read the entire set of problems first; mark the ones you know how to solve and cross out the ones you don't.
4. Do exactly four problems. No bonus points will be given to a fifth solution and beyond. If you have extra time, double check your work.

–Amin Witno