Exam 1 **Abstract Algebra 2** **22–03–2009**

Choose any 4 problems from the following 6 problems.

1. (a) What is the definition of a ring R? (b) Let $a \in R$. Prove that the set $S = \{x \in R \mid xa = ax\}$ is a subring of R.

2. (a) What is the definition of a field? (b) Prove that the set $S = \{a + b\sqrt{5} \mid a, b \in \mathbf{Q}\}$ is a subfield of the rational number field $\mathbf{Q}$.

3. (a) What is the definition of an integral domain? (b) Prove that a finite integral domain is a field.

4. Let R be a commutative ring with unity. (a) What is the definition of a unit element? (b) Prove that the set $S = \{x \in R \mid x \text{ is a unit}\}$ is a group under multiplication.

5. (a) What is the definition of an ideal? (b) Prove that the set $S = \{0, 3\}$ is an ideal of $\mathbf{Z}_6$.

6. Let I be an ideal of a ring R. (a) What is the definition of the factor ring R/I? (b) Prove that if R is a field with n elements, then R/I has either 1 or n elements.

Notes:

1. Full credit will only be given to a solution which is logically correct. Be very careful in what you write!

2. You may assume all the theorems given in the notes, unless when the problem asks you to prove the theorem.

3. Do not spend too much time on a single problem. Read the entire set of problems first; mark the ones you know how to solve and cross out the ones you don't.

4. Do exactly four problems. No bonus points will be given to a fifth solution and beyond. If you have extra time, double check your work.

–Amin Witno