Philadelphia University

Department of Basic Sciences

Final Exam **Abstract Algebra 2** **20–01–2009**

Part I. Choose one answer.

1. Which ring is not a field?
   (a) Z_{13}
   (b) $Z_3 \times Z_5$
   (c) $\{a + b\sqrt{2} \mid a, b \in Q\}$
   (d) Q
2. Which element of Z_{24} is a zero divisor?
   (a) 5
   (b) 23
   (c) 1
   (d) 15
3. Which ring is an integral domain?
   (a) Z_9
   (b) $Z_3 \times Z_5$
   (c) $\{a + bi \mid a, b \in Z\}$
   (d) $M_2(Z)$
4. Which polynomial is irreducible over Z_{11}?
   (a) $x^4 + 1$
   (b) $x^2 - 5$
   (c) $x^2 + x + 4$
   (d) $x^3 - 2x^2 + 2$
5. Over which field is $x^2 + 1$ reducible?
   (a) R
   (b) Z_3
   (c) Z_7
   (d) Z_{13}

6. The ring $Q[x]$ is a field.
   (a) True
   (b) False
7. Which element is not algebraic over Q?
   (a) i
   (b) $\sqrt{2}+\sqrt{3}$
   (c) $\sqrt{2}+\sqrt[3]{2}$
   (d) all the above are algebraic
8. Given $Q(\sqrt{2},\sqrt{3}) = Q(a)$. Then $a =$
   (a) $\sqrt{6}$
   (b) $\sqrt{2}+\sqrt{3}$
   (c) $\sqrt[4]{6}$
   (d) $\sqrt[3]{2}$
9. $[Q(\sqrt[4]{3},\sqrt[6]{5}) : Q] =$
   (a) 24
   (b) 12
   (c) 10
   (d) 2
10. The factor ring $Q[x]/(f)$ is not a field when
   (a) $f(x) = x^2+1$
   (b) $f(x) = x^2+2$
   (c) $f(x) = x^2-7$
   (d) $f(x) = x^2-x-2$

Part II. Write complete solution.

1. Prove that the ring Z_n is a field if and only if n is a prime number.
2. Find the minimal polynomial of $\sqrt{2}+5\sqrt{7}$ over Q.
3. Prove that every ideal in $Q[x]$ is principal.
4. Let R be a commutative ring and $a \in R$. Prove that the set $I = \{r \in R \mid ar = 0\}$ is an ideal of R.
5. Show that x^3+4x^2+4x+1 is reducible over $\mathrm{Z}_5[x]$ and then factor it into irreducible polynomials.

–Amin Witno